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IR Dynamics of $d = 2$, $\mathcal{N} = (4,4)$ gauge theories and DLCQ of "little string theories"

Ofer Aharony

Department of Physics and Astronomy
Rutgers University, Piscataway, NJ 08855
E-mail: `oferah@physics.rutgers.edu`

Micha Berkooz

Department of Physics, Jadwin Hall
Princeton University, NJ 08544
E-mail: `mberkooz@feynman.princeton.edu`

Abstract: We analyze the superconformal theories (SCFTs) which arise in the low-energy limit of $\mathcal{N} = (4,4)$ supersymmetric gauge theories in two dimensions, primarily the Higgs branch SCFT. By a direct field theory analysis we find a continuum of "throat"-like states localized near the singularities of the Higgs branch. The "throat" is similar to the "throat" found in the Coulomb branch of the same theories, but the full superconformal field theories of the two branches are different. A particular example is the SCFT of the $\mathbb{R}^4/\mathbb{Z}_2$ sigma model with zero theta angle. In the application of the Higgs branch SCFTs to the DLCQ description of "little string theories" (LSTs), the "throat" continuum is identified with the continuum of "throat" states in the holographic description of the LSTs. We also match the descriptions of the string interactions (in the "throat" region) in the DLCQ and holographic descriptions of the $\mathcal{N} = (2,0)$ LSTs.

Keywords: Field Theories in Lower Dimensions, Sigma Models, Conformal Field Models in String Theory, Supersymmetric Effective Theories.

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Contents

1. Introduction

Gauge theories with eight supercharges in various dimensions have been an interesting subject for several years now, mostly because supersymmetry severely restricts the quantum corrections in these theories and allows many exact computations to be made. The two dimensional case is especially interesting because these gauge theories appear in linear sigma model descriptions of some string theory compactifications and in discrete light-cone (DLCQ) descriptions of "little string theories" (LSTs).

The moduli space of gauge theories with eight supercharges (below 6 dimensions) has two general types of branches: the Coulomb branch, where the scalars in the vector multiplets obtain vacuum expectation values, and the Higgs branch, where the scalars in the hypermultiplets obtain vacuum expectation values. To the extent that the metric on the moduli space is a useful notion (i.e. it is the leading term in a systematic expansion), it is very much constrained by supersymmetry. In theories with 8 supercharges, supersymmetry forbids any quantum corrections to the Higgs branch moduli space metric. On the Coulomb branch, in perturbation theory only one-loop corrections to the metric are possible, and often the non-perturbative corrections are also understood (or forbidden altogether). Thus, it is generally possible to compute exactly the metric on the moduli space in these theories. Naively, this means that we have good control over the low-energy behavior in these theories. However, there are some cases where the moduli space approximation breaks down, and more information is needed to give a full low-energy description. Although this break-down can happen in a variety of ways, which are generally indicated by higher-order terms in the effective action becoming important, it certainly occurs when the metric becomes negative, and often also when it is singular.

In gauge theories with 8 supercharges, such singularities of the metric occur near the meeting points of the Higgs and Coulomb branches, where additional degrees of freedom classically (and in some cases also quantum mechanically) become massless. The significance of the singularities changes in different dimensions. Above two dimensions, there is a real moduli space of vacua, and once a point on the moduli space is chosen the theory is well-described (at low enough energies) by fluctuations on the moduli space around that point. Away from singular points in the moduli space the theory is typically free in the IR, while at singular points it is typically described by a non-trivial superconformal field theory (SCFT). In two or less dimensions the situation is different, since there is no real moduli space of vacua due to quantum fluctuations (the moduli space metric is still often relevant as part of a Born-Oppenheimer approximation). Singularities in the metric indicate that the sigma model approximation breaks down (although one can sometimes still obtain sensible answers for a carefully chosen restricted class of questions).

In this paper we will discuss the $1+1$ dimensional superconformal field theories which are the low-energy limit of $\mathcal{N}=(4,4)$ supersymmetric gauge theories in two dimensions. The low-energy dynamics in this case is described by two superconformal field theories, one corresponding to the Higgs branch and one to the Coulomb branch; the two theories decouple at low energies [1, 2, 3]. The moduli space metrics of the two IR theories have singularities (and can even become negative on the Coulomb branch), indicating that higher order terms are important, and one requires a different description of the theory near the singular regions. In particular, there are various reasons to believe that there is a continuous spectrum associated with the singularities in the Higgs branch (one example of such a singularity is the $\mathbb{R}^4/\mathbb{Z}_2$

singularity with zero theta angle). In this paper we will develop, following [3, 4], a field theory method which yields a useful description of the behavior of the theory near the singularities. This is done by means of a simple effective field theory which explicitly exhibits the continuous spectrum, and in some cases also the leading interaction term in a systematic expansion.

The method we use involves an explicit lagrangean for the Higgs branch SCFT, in which the Coulomb branch fields appear as auxiliary variables. We can then integrate out the hypermultiplets to obtain an effective action for these auxiliary fields. This method was outlined in [4]. The main difference will be that whereas there the hamiltonian formulation was emphasized (which is more convenient in the case of quantum mechanics), here we will focus on the lagrangean approach, and we will take advantage of the power of $1+1$ dimensional superconformal symmetry. The approach we use is also similar to that of [5], although the interpretation is quite different, and the methods we use are similar to those of [6].

In our description the singularity will be replaced by a semi-infinite "throat",[1] as in the Coulomb branch when the one-loop corrections are taken into account [9]. A similar "throat" was found [10] in the analysis of D1-D5 theories which have the same singularities as the Higgs branch theories we discuss here; however, their analysis used the AdS/CFT correspondence and a large N approximation, while our analysis will be directly in the field theory and valid for any value of N. Furthermore, our method is applicable for any Higgs branch of a $\mathcal{N}=(4,4)$ theory (although we will focus on U(N) gauge theories, which are related to the D1-D5 system), and we will discuss both the R-R and the NS-NS sectors of the theory.

We will focus mainly on the U(N) gauge theory with an adjoint hypermultiplet and N_f fundamental hypermultiplets (which arises as the low-energy theory on D1-branes inside D5-branes in flat space). One of our motivations for studying this theory is to examine its usefulness as a DLCQ description [2, 3] of "little string theories" (LSTs) with 16 supercharges [11, 12, 13]. The "little string theories" are known from their holographic description (which includes a linear dilaton background) to have a continuous spectrum above some mass gap [14, 15]. We will show that in the DLCQ this continuous spectrum may be identified with the continuous spectrum appearing at the singularities in the corresponding SCFTs, and that we can also reproduce the string interactions in the weakly coupled region of the holographic description.

The organization of the paper is as follows. In section 2 we review $\mathcal{N}=(4,4)$ gauge theories in two dimensions and their low-energy limits,[2] and we give an explicit

[1]For the case of $\mathbb{R}^4/\mathbb{Z}_k$ singularities embedded in a space with a compact circle transverse to the singularity, there is a T-duality relating them to an NS5-brane configuration which has a similar semi-infinite "throat" [7, 8].

[2]These theories were analyzed using brane constructions in [16], [17], and were argued to be dual to the dimensional reduction of some non-trivial $2+1$ dimensional field theories in [18].

lagrangean description for the conformal theory of the Higgs branch which is also useful near the singularities. In section 3 we analyze the behavior of this theory near the singularities of the Higgs branch in $U(N)$ gauge theories, and show that it develops a "throat" region which can be explicitly exhibited and analyzed. In section 4 we analyze the DLCQ of the $(2,0)$ LSTs, which corresponds to a particular Higgs branch SCFT. We match the continuous part of the spectrum to that which arises in the linear dilaton region of the holographic description of the LSTs, and we also reproduce the string interactions there. In section 5 we analyze the DLCQ of $(1,1)$ LSTs [18, 19], which is a particular Coulomb branch SCFT. In this case we can easily exhibit the low-energy states of the $(1,1)$ LSTs (which are free W-bosons), but it is more difficult to obtain a precise understanding of the continuum states since the "throats" of the relevant Coulomb branch theories are less well understood. We end in section 6 with a summary and some remarks.

2. Two dimensional $\mathcal{N} = (4,4)$ gauge theories and their low energy limits

2.1 $\mathcal{N} = (4,4)$ gauge theories

Gauge theories with $\mathcal{N} = (4,4)$ supersymmetry in two dimensions may be viewed as the dimensional reduction of four dimensional $\mathcal{N} = 2$ gauge theories, or of six dimensional $\mathcal{N} = (1,0)$ gauge theories. Like the other theories with eight supercharges, their matter content includes vector multiplets in the adjoint representation of the gauge group and hypermultiplets which can be in arbitrary representations of the gauge group. The six dimensional gauge theories have an $SU(2)_R$ R-symmetry. Upon dimensional reduction to two dimensions there is an additional $SO(4) \simeq SU(2)_r \times SU(2)_l$ symmetry acting on the four reduced dimensions. This is also an R-symmetry since the supercharges are a spinor of this $SO(4)$ group; the left-moving (positive chirality) supercharges are in the $(\mathbf{2},\mathbf{1},\mathbf{2})$ representation of $SU(2)_l \times SU(2)_r \times SU(2)_R$ while the right-moving (negative chirality) supercharges are in the $(\mathbf{1},\mathbf{2},\mathbf{2})$ representation.

The matter content of the vector multiplets and the hypermultiplets may be easily derived using the dimensional reduction from six dimensions, and it is detailed in table 1.

The parameters of $\mathcal{N} = (4,4)$ gauge theories in two dimensions include the gauge coupling (or gauge couplings for non-simple gauge groups), Fayet-Iliopoulos terms and theta angles for abelian components of the gauge group, and masses for the hypermultiplets. We will discuss here mostly the case where the Fayet-Iliopoulos terms, theta angles and masses all vanish, so that the only parameter is the gauge coupling constant g_{YM}, whose scaling dimension is $[g_{YM}] = 1$. Because of this scaling dimension, the gauge theories become free in the UV, but are strongly coupled in

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		$\mathrm{SU}(2)_l$	$\mathrm{SU}(2)_r$	$\mathrm{SU}(2)_R$	
vector multiplet:	A_μ	1	1	1	vector
	V	2	2	1	real scalar
	ψ_V^L	1	2	2	left moving fermion
	ψ_V^R	2	1	2	right moving fermion
hypermultiplet:	H	1	1	2	complex scalar
	ψ_H^L	2	1	1	complex left moving fermion
	ψ_H^R	1	2	1	complex right moving fermion.

Table 1: Field content of $\mathcal{N} = (4,4)$ supersymmetry multiplets.

the IR, where the perturbative gauge theory description is not valid. Schematically, the gauge theory lagrangean (suppressing all indices) is given by

$$\mathcal{L} = \int d^2x \biggl\{ \frac{1}{4g_{YM}^2} \mathrm{tr}\left[F_{\mu\nu}^2 + (\mathcal{D}_\mu V)^2 + [V,V]^2 + \bar{\psi}_V\gamma^\mu\mathcal{D}_\mu\psi_V + \bar{\psi}_V[V,\psi_V] + D^2\right] + \\ + \sum_{hypers} \left[(\mathcal{D}_\mu H)^2 + (VH)^2 + \bar{\psi}_H\gamma^\mu\mathcal{D}_\mu\psi_H + \bar{\psi}_H V\psi_H + \bar{\psi}_V H\psi_H + DHH\right]\biggr\}, \quad (2.1)$$

where D are the three auxiliary fields in the vector multiplet (in the **3** of $\mathrm{SU}(2)_R$) and $\mathcal{D}_\mu \simeq \partial_\mu + A_\mu$ is the covariant derivative. The scaling dimensions of the various fields in this lagrangean are $[A_\mu] = [V] = 1$, $[\psi_V] = 3/2$, $[D] = 2$, $[H] = 0$ and $[\psi_H] = 1/2$.

The classical moduli space of these theories, like that of other theories with eight supercharges (below six dimensions), includes two types of configurations: the Coulomb branch, where the scalars in the vector multiplets obtain vacuum expectation values (VEVs), and the Higgs branch, where the scalars in the hypermultiplets obtain VEVs (in general there are also mixed branches but they do not seem to raise any new issues so we will limit ourselves here to a discussion of the two extreme cases). Of course, quantum field theories in two dimensions do not actually have a moduli space of vacua, since wave functions tend to spread out on the classical moduli space; hence, one needs to treat the IR limit more carefully.

2.2 Basic properties of the IR limit(s)

The low-energy (IR) limit of the gauge theory involves taking $g_{YM} \to \infty$, and one expects to obtain in this limit an $\mathcal{N} = (4,4)$ superconformal field theory (SCFT).[3] However, it is believed [1, 22, 2, 3] that the low-energy limit is in fact two decoupled superconformal field theories, one describing the Higgs branch of the theory and the other corresponding to the Coulomb branch; for finite g_{YM} wave functions can

[3] The IR theory is (by definition) a scale-invariant theory. For theories with a compact moduli space this implies [20, 21] that it is also a conformally invariant theory; we expect this to be true also in our case, even though the moduli space is non-compact.

spread from one branch to the other, but it is believed that in the $g_{YM} \to \infty$ limit the distance between the branches goes to infinity and they decouple.

The simplest general argument for this [3] comes from an analysis of the R-symmetries of the IR $\mathcal{N} = (4,4)$ SCFT. The $\mathcal{N} = (4,4)$ superconformal algebra includes left-moving and right-moving SU(2) Kac-Moody algebras. Both the Higgs and the Coulomb branches have semi-classical regions for large VEVs, whose description includes scalar fields with a flat metric; a symmetry which rotates such scalar fields cannot be separated into left-moving and right-moving pieces, so it cannot be part of the R-symmetry of the IR SCFT. This determines that in a SCFT of the Higgs branch $\mathrm{SU}(2)_R$ cannot be (part of) an R-symmetry, since the scalars in the hypermultiplets are charged under this group. Similarly, $\mathrm{SU}(2)_r \times \mathrm{SU}(2)_l$ cannot be an R-symmetry in the theory of the Coulomb branch. It is natural to expect that the R-symmetry of the Higgs branch is exactly $\mathrm{SU}(2)_r \times \mathrm{SU}(2)_l$, while the R-symmetry of the Coulomb branch is an $\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2$ symmetry which is not visible in the gauge theory but which has $\mathrm{SU}(2)_R$ as its diagonal subgroup. Since the Coulomb and Higgs branch superconformal theories have different R-symmetries it is clear that they cannot be identified. In the Higgs branch SCFT we can use the identification of the SU(2) R-symmetries and the relation between their chiral anomaly and the central charge to compute the central charge, $c = 6(n_H - n_V)$ where n_H is the number of hypermultiplets and n_V is the number of vector multiplets [3].

As discussed in [3], the Higgs and Coulomb branch theories have different scaling dimensions for the various fields. In the Higgs branch the hypermultiplets obtain VEVs so $[H] = 0$, while in the Coulomb branch the vector multiplet scalars obtain VEVs so $[V] = 0$. In the Higgs branch theory, the scaling dimensions of the various fields agree with their classical scaling dimensions in (2.1); the fact that $[V] = 1$ follows from the fact that it is in the $(\mathbf{2}, \mathbf{2})$ representation of the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ R-symmetry group in the $\mathcal{N} = (4,4)$ superconformal algebra. Thus, we claim (following [4, 3]) that the exact lagrangean for the Higgs branch SCFT is obtained by the naive limit of $g_{YM} \to \infty$ in (2.1), which removes the first line and leaves us only with the second line:

$$\mathcal{L}_H = \int d^2x \sum_{hypers} \Big\{ (\mathcal{D}_\mu H)^2 + (VH)^2 + \bar{\psi}_H \gamma^\mu \mathcal{D}_\mu \psi_H + \bar{\psi}_H V \psi_H + \bar{\psi}_V H \psi_H + DHH \Big\} . \tag{2.2}$$

In this limit the kinetic terms of the vector multiplet fields all vanish, so they become auxiliary fields.

One description of the Higgs branch SCFT is as a supersymmetric sigma model on the Higgs branch moduli space: this is recovered if one integrates out the auxiliary fields in (2.2). This can be done explicitly since the action is quadratic in these fields. The integration over the D fields and the gauge fields gives the constraints which limit the configuration space to the classical Higgs branch, while the integration over V gives rise to the 4-fermi interactions of the supersymmetric non-linear sigma model. However, following [4], in order to describe the theory near the singularities in the moduli space it is more useful to regard the vector multiplet fields, which (using (2.2)) are composite operators on the Higgs branch of the form $V \sim \psi_H^L \psi_H^R / H^2$, as the basic objects, and to perform a change of variables to these new coordinates. Note that these variables are all invariant under any global symmetries acting on the hypermultiplets, so we cannot describe states which are charged under such global symmetries in terms of the new variables.

In the lagrangean approach this procedure amounts to integrating over the hypermultiplet fields and inducing an effective action for the vector multiplet fields [3]. This is basically the same technique as in the $\mathbb{CP}^{N-1}$ model (described in [23] and references therein). Obviously, in this approach we are throwing away most of the dynamical degrees of freedom of the theory, and naively one might expect the resulting (non-local) effective action to be uncontrollable. However, since a non-zero value of V gives a mass to H and ψ_H, there is actually a systematic expansion in E/V (where E is the energy), which gives a good description of the states which are localized near the singularities. This will be carried out in section 3, and used for DLCQ applications in section 4.

When is this description expected to be valid? Using (2.2) we can think of V^2 as the mass squared of the hypermultiplets, so we expect such a description to be valid for energies below this mass scale. This is just the usual Born-Oppenheimer approximation. Alternatively, the effective action for the vector multiplets may be expanded in a power series in the dimensionless dV/V^2, and we can neglect the higher order terms in this expansion when $dV \ll V^2$. Thus, we expect to get a good description in terms of the vector multiplets for low-energy wave functions concentrated around large values of V. If we integrate out the vector multiplets instead of the hypermultiplets, we find that V is equal to a bilinear in the hypermultiplet fermions divided by H^2, so in some sense large values of V correspond to small values of H, close to the singularity of the Higgs branch.[4] Therefore, we expect the effective vector multiplet theory to give a good description of the region near the singularities of the Higgs branch.

If, alternatively, we want to focus on the Coulomb branch theory, we would like to have $[V] = 0$, so we need to take $V \to \infty$ as $g_{YM} \to \infty$ keeping the dimensionless $\Phi \equiv V/g_{YM}$ constant. This will be the coordinate on the moduli space of the Coulomb branch SCFT. Note that this means that any finite value of Φ in the Coulomb branch theory corresponds to an infinite V from the point of view of the original gauge theory and of the Higgs branch theory, which is consistent with the infinite separation between the two branches in the IR. It is not clear whether the same procedure we used for analyzing the Higgs branch can also be used for the Coulomb branch,

[4]Related observations were made in [24].

i.e. whether we can treat the hypermultiplets as auxiliary variables in the Coulomb branch SCFT and use them to understand its singularities. We have not been able to write down an explicit lagrangean for the Coulomb branch SCFT, analogous to (2.2). The approach we will use to analyze the Coulomb branch SCFT in section 5 will be to first integrate out the hypermultiplets for finite g_{YM}, and then take $g_{YM} \to \infty$ with the scaling which keeps Φ finite. This scaling keeps both the tree-level and the one-loop terms in the Coulomb branch moduli space metric finite.

3. The "throat" of the Higgs branch

The moduli space metric on the Higgs branch does not receive any quantum corrections. Thus, unlike the situation in the Coulomb branch, which we will discuss in section 5, the singularities in the Higgs branch (where classically it meets the Coulomb branch) remain a finite distance away. Nevertheless, there are many reasons to believe that the moduli space description breaks down near the singularities [1]. A special case of such a singularity, which arises in the U(1) theory with two hypermultiplets, is the $\mathbb{R}^4/\mathbb{Z}_2$ singularity with zero theta angle. When this SCFT is used for string theory compactifications, some correlation functions diverge, as is evident from the existence of light non-perturbative states in space-time. In [10] it was claimed, using the AdS/CFT correspondence [25, 26, 27, 28], that at the singularities the Higgs branch develops a semi-infinite "throat" in appropriate variables. Such a "throat" (which appears also in the perturbative analysis of the Coulomb branch metric [9]) leads to a continuous spectrum of dimensions of primary operators (beyond the continuum associated with the classical non-compactness of the moduli space) and to divergences in various correlation functions. In this section we will discuss a field theory analysis of the singularities which, among other things, reproduces this claim directly (without using a large gauge group approximation as in [10]), as the leading term in a systematic expansion.

3.1 The "throat" in the abelian case

Let us begin by analyzing the superconformal theory corresponding to the Higgs branch of a U(1) gauge theory with N_f charged hypermultiplets, with $N_f > 1$ (otherwise there is no Higgs branch). This is an $\mathcal{N} = (4,4)$ SCFT with central charge $c = 6(N_f - 1)$. As explained before, we would like to obtain an effective description of the singularity in terms of the vector multiplet fields. This effective theory is expected to give a good description of some states which are localized near the singularities. We begin with the case of a U(1) gauge field as the easiest implementation of our approach, and discuss the additional features of the non-abelian U(N) gauge theory in the next subsection. The U(1) case is also interesting in its own right since, as mentioned above, the $N_f = 2$ Higgs branch theory gives the $\mathbb{R}^4/\mathbb{Z}_2$ sigma model with zero theta angle.

The effective theory We would like to integrate out the fundamental hypermultiplets and obtain an effective action for the vector multiplet fields. As explained above, the appropriate expansion is in terms of dV/V^2. The leading terms in this expansion, which are the metric term and its supersymmetric partners, are known because of supersymmetry, and they are expected to dominate at energies much smaller than V. Supersymmetry and SO(4) symmetry [22, 9] constrain the metric for the four scalar fields in the vector multiplet to be of the form

$$ds^2 = \left(a + \frac{b}{2|V|^2}\right) dV^2 \,. \tag{3.1}$$

Since $[V] = 1$, a is dimensionful and so must be zero in the superconformal theory we are discussing. On the other hand, b can be non-zero, and a 1-loop computation using either the original action or the action (2.2) gives[5] $b = N_f$. In the original theory dimensional analysis determines that b cannot depend on g_{YM} so the one-loop result for b must be exact; we claim that this is true also in the IR theory (2.2). Similarly, a torsion term also arises at one-loop [9].

Together, these terms lead to a "throat" metric. Changing to radial coordinates $dV^2 = dv^2 + v^2 d\Omega_3^2$, and defining a new variable $\phi = \sqrt{N_f/2}\log(v/M)$ for some mass scale M, we find a sigma model with metric

$$ds^2 = d\phi^2 + \frac{N_f}{2} d\Omega_3^2 \tag{3.2}$$

and torsion

$$H = -N_f d\Omega \,, \tag{3.3}$$

where $d\Omega_3^2$ is the metric and $d\Omega$ is the volume form on the 3-sphere. The metric and torsion on the S^3 coordinates give exactly (after adding in the fermions) a level N_f supersymmetric SU(2) WZW model. From the construction it is clear that the left-moving and right-moving SU(2) symmetries of this model are part of the $\mathrm{SU}(2)_r \times \mathrm{SU}(2)_l$ symmetries (acting on V) that are in the superconformal algebra; we will discuss the exact form of the SU(2) currents below.

However, the description above cannot be the whole story. The simplest way to see this is to note that it does not have the correct central charge, $c = 6(N_f - 1)$. The "throat" theory describes the region of very large ϕ, where it differs from the full superconformal theory only by higher derivative terms that do not contribute to the conformal anomaly, so it must have the same conformal anomaly as the full theory. We would like to argue that the central charges are matched by having in the effective lagrangean a background charge for ϕ, of the form $\mathcal{L} \sim \frac{Q}{8\pi}\phi\sqrt{g}\mathcal{R}$ (where

[5] It is clear that b must be proportional to N_f, since the whole induced action for the vector fields is proportional to N_f, arising from integrating over N_f independent hypermultiplets. This form of the metric also follows (up to a constant) from conformal invariance, since $[V] = 1$.

$\mathcal{R}$ is the scalar curvature of the background 2-dimensional metric g), with

$$Q = (N_f - 1)\sqrt{\frac{2}{N_f}}\,. \tag{3.4}$$

The arguments for this are the following:

1. In the original action (2.2), a conformal transformation takes $V \to e^{\alpha} V$ (along with some action on the fermions and the auxiliary fields), and is known to change the action by $\frac{c}{48\pi}\alpha\sqrt{g}\mathcal{R} = \frac{N_f-1}{8\pi}\alpha\sqrt{g}\mathcal{R}$. After the change of variables, this transformation acts on ϕ as $\phi \to \phi + \alpha\sqrt{N_f/2}$; thus, to get the correct variation of the action, it must include a term of the form $\frac{N_f-1}{8\pi}\sqrt{2/N_f}\phi\sqrt{g}\mathcal{R}$, giving (3.4). This is nothing but the usual argument of adding a background charge to obtain the correct central charge. Using the fact that the supersymmetric level N_f SU(2) WZW model may be written as the sum of a bosonic level (N_f-2) SU(2) WZW model plus three free fermions, we find that in order to have the correct central charge we require

$$6(N_f - 1) = 2 + \frac{3(N_f-2)}{N_f} + (1 + 3Q^2)\,, \tag{3.5}$$

giving $Q = (N_f - 1)\sqrt{2/N_f}$ (as in [9], the positive sign is determined by identifying the strong coupling region with the region near the singularity).

2. The "throat" theory (ignoring all the higher order corrections) in fact has a large $\mathcal{N} = 4$ algebra [29, 30, 31, 32], as reviewed for example in section 4 of [10]. This large $\mathcal{N} = 4$ algebra has two superconformal $\mathcal{N} = 4$ subalgebras, one with $c = 6$ and the other with $c = 6(N_f - 1)$. The first algebra appears in the "throat" which is found in the Coulomb branch [9]; in this algebra, the SU(2) currents (of level 1) do not involve the bosonic SU(2) WZW model but only the free fermionic fields (recall that these are, for the left-movers, in the $(\mathbf{1},\mathbf{2},\mathbf{2})$ representation of $SU(2)_l \times SU(2)_r \times SU(2)_R$, and that $SU(2)_R$ is related to the superconformal algebra of the Coulomb branch). However, it is the other $\mathcal{N} = 4$ subalgebra which is relevant for the Higgs branch, for example because it includes the $SU(2)_r \times SU(2)_l$ currents of level $(N_f - 1)$ which are in the superconformal algebra of the Higgs branch (in the "throat" theory these currents are the sum of the bosonic level $(N_f - 2)$ currents and a level one current from the free fermions). The difference between the two subalgebras involves exactly a shift in the energy-momentum tensor which changes the background charge of the ϕ field from the value $\tilde{Q} = -\sqrt{2/N_f}$ of the Coulomb branch "throat" [9] to the value $Q = (N_f - 1)\sqrt{2/N_f}$ of the Higgs branch "throat".

Thus, we conclude that the effective theory in the region of large V is given by four free fermions, a free scalar with background charge $Q = (N_f - 1)\sqrt{2/N_f}$, and a level $(N_f - 2)$ bosonic SU(2) WZW model. For $N_f = 2$ this "throat", with its continuum of states (given by states with any momentum in the ϕ direction), is part of the $\mathbb{R}^4/\mathbb{Z}_2$ sigma model with zero theta angle.

Relation to the Coulomb branch throat The theory we found in the "throat" seems very similar to the theory found in the Coulomb branch "throat" of the same gauge theories [9], except for the different background charge for the scalar.[6] This similarity should not be surprising, since before we take the low-energy limit the two "throats" are actually connected to each other, and we can have states which start out as some wave packet (with energy well below g_{YM}) in the Coulomb branch moving towards the singularity, then enter the "throat" and eventually come out of the "throat" into the Higgs branch. This suggests that there should be a one-to-one mapping between the states propagating in the "throat" regions of the two theories, although there is no direct relation between the "throats" after taking the IR limit. Studying this mapping will lead us to discover an extra vacuum energy in the "throat" of the Higgs branch SCFT on a cylinder (in the RR sector).

Let us look at the finite coupling gauge theory on a cylinder with periodic boundary conditions for the fermions (the RR sector). At energies well below the Yang-Mills coupling (but still above a mass gap which will be discussed shortly, which is a number times the inverse radius of the circle), there are four distinct regions in the moduli space:

1. The asymptotic flat region of the Coulomb branch.
2. The "throat" of the Coulomb branch, which includes a scalar with background charge $\tilde{Q} = -\sqrt{2/N_f}$.
3. The "throat" of the Higgs branch, which includes a scalar with background charge $Q = (N_f - 1)\sqrt{2/N_f}$.
4. The asymptotic region of the Higgs branch.

We would like to compute the energy gap between the lowest states in regions 3 and 4, which become part of the Higgs branch SCFT when we take the low-energy limit. In all regions there is no normalizable ground state, but there are delta-function normalizable states whose energy is bounded from below, and we will refer to this lower bound as the "ground state" energy of the appropriate region.

We begin by reviewing the energies of states in general "throat" (linear dilaton) CFTs. In a "throat" region, if we normalize the state corresponding to the identity

[6] Corrections to the "throat" in the two cases will be very different, occurring at small V for the Higgs branch "throat" and at large V for the Coulomb branch "throat".

operator to have zero energy, then the energy of a state with momentum q, corresponding to an operator $e^{iq\phi}$, is given by $E = q(q+iQ)$ (in units determined by the radius of the cylinder). In order to have a real energy we require $q = -iQ/2+q_0$ where q_0 is real, and then $E = q_0^2 + Q^2/4$, leading to an energy gap ("ground state" energy) of $Q^2/4$ for states propagating in the "throat" (and a continuum above this gap). Of course, "throat" states can also include some operators involving the fermions and the level $(N_f - 2)$ WZW model, but we will ignore these for now (since these sectors are the same in the Coulomb branch and Higgs branch "throats").

We will now compute the "ground state" energies in regions 1, 2 and 4, and then by matching the "throat" states we will compute the "ground state" energy in region 3. In regions 1 and 4 supersymmetry tells us that the "ground state" energies (in the RR sector) vanish. In region 2 the theory is just a "throat" theory [9] given in terms of the original Coulomb branch variables (with no additional energy shifts), so the "ground state" energy there is $\tilde{Q}^2/4$. We claim that this can be identified with the "ground state" energy in the Higgs branch "throat", since states going into one come out of the other (the scalar ϕ in the "throat" theory is free, and this approximation becomes better and better "down the throat"). Thus, we conclude that the Higgs branch "throat" has a constant contribution $\tilde{Q}^2/4 - Q^2/4$ to its vacuum energy, so that the hamiltonian for the Higgs branch "throat" is given by (ignoring the fermions and the WZW model)

$$H = q(q+iQ) + \frac{(\tilde{Q}^2 - Q^2)}{4}\,. \tag{3.6}$$

Using this hamiltonian, any state corresponding to an operator $e^{i\tilde{q}\phi}$ and a wave-function $e^{i(\tilde{q}+i\tilde{Q}/2)\phi}$ in the Coulomb branch "throat" can be mapped into a state with the same wave-function and energy in the Higgs branch "throat", corresponding to the operator $e^{iq\phi}$ where $q + iQ/2 = \tilde{q} + i\tilde{Q}/2$.

The discussion in the previous paragraph may appear to be at odds with the discussion of [10], where it was argued that the Higgs branch "throat" *does* have a mass gap of $Q^2/4$ (and not $\tilde{Q}^2/4$ as we find above) above the "ground state" energy in region (1). However, the discussion of [10] was in the NS-NS sector of the superconformal field theory and not in the R-R sector which we analyzed above (in the AdS_3/CFT_2 correspondence, the NS-NS sector vacuum is identified with the AdS_3 vacuum, while the R-R vacuum is identified with the BTZ black hole). The vacuum ("ground state") energy of the NS-NS sector is shifted compared to the R-R sector by $(-c/12)$. Thus, if we were to repeat the same analysis we did above in the NS-NS sector, we would find that the Coulomb branch "throat" states would have $E = \tilde{q}(\tilde{q} + i\tilde{Q}) - c_C/12$, which means that when they exit from the Higgs branch "throat" they would have an energy of

$$E = \tilde{q}(\tilde{q} + i\tilde{Q}) + \frac{(c_H - c_C)}{12}\,, \tag{3.7}$$

over the Higgs branch vacuum (the "ground state" energy of region 4). The mass gap in the Higgs branch "throat" in this sector should thus be

$$\frac{\tilde{Q}^2}{4} + \frac{c_H - c_C}{12} = \frac{1}{2N_f} + \frac{(6N_f - 6) - 6}{12} = \frac{Q^2}{4}\,, \tag{3.8}$$

which is the naive mass gap of the Higgs branch. Thus, in this sector we do not need any vacuum energy to match the Coulomb branch "throat" to the Higgs branch "throat"; the vacuum energy we found disappears when we do the appropriate shift relating the R-R and NS-NS sectors. Therefore, our results are in perfect agreement with those of [10]. In the R-R sector of the Higgs branch SCFT the mass gap (the energy difference between the "ground state" and the continuum) is $1/2N_f$, while in the NS-NS sector it is $(N_f - 1)^2/2N_f$.

3.2 The "throat" in the non-abelian case

For $N > 1$, the exact analysis of the "throat" becomes more complicated. However, we still expect integrating out the hypermultiplets to lead to an effective action for the vector multiplets which is similar to the action we got in the abelian case; gauge invariance and dimensional analysis severely limit the possible terms that can appear.

The term which we would like to focus on in the non-abelian case is a "commutator" term, i.e. a potential term which is non-zero when the V matrices do not commute. We will not discuss this term in general, but only in the vicinity of the point in configuration space in which the V matrices are all proportional to the identity matrix (this case will be the most interesting for the DLCQ application discussed below). Note that in general this potential term suggests that the dominant configurations in the IR are such that the V matrices commute (and can be taken to be diagonal). The computation of the potential and the effective theory under such more general circumstances is also straightforward, but we will not discuss it here.

Let us expand the matrix V as

$$V = V_0 \left(I + \sqrt{\frac{2}{N_f}} \delta V \right), \tag{3.9}$$

where I is the $N \times N$ identity matrix and we normalized δV so it will have a canonical kinetic term in the "throat". Now, we have a double expansion in δV and in $d(\delta V)/V_0$, and the leading potential term in this expansion is of the form

$$N_f \frac{\mathrm{tr}([V,V]^2)}{2V_0^2} = \frac{2}{N_f} V_0^2 \,\mathrm{tr}([\delta V, \delta V]^2)\,. \tag{3.10}$$

This term is related by SUSY to the kinetic term $N_f \,\mathrm{tr}(dV)^2/2V_0^2$ which we can obtain as in the abelian case, but we can also compute it directly. The mass squared matrix for the bosonic elements of the hypermultiplets is a matrix of the form V^2, while for the fermionic elements we find matrices of the form $V^2 + i[V, V]$ (with appropriate SO(4) indices). The one-loop computation of the vacuum energy measures the difference in energy between the ground states of the bosons and fermions in the hypermultiplet, and we find exactly the potential (3.10).

If we have only fundamental hypermultiplets then generally, for $N > 1$, a moduli space approximation is not expected to be valid using the vector multiplet variables,[7] since integrating out the off-diagonal vector multiplets gives rise to a negative metric whenever two eigenvalues approach each other [9]. As long as we keep the full vector multiplet matrices (as above) this problem does not arise. However, we do not fully understand the IR dynamics of these vector multiplets. We expect a moduli space approximation to be valid only when all the eigenvalues of V are far separated from each other.[8]

3.3 The ADHM sigma model

For applications described later in the paper to "little string theories" it will be useful to analyze also theories with an adjoint hypermultiplet, in addition to the N_f multiplets in the fundamental representation of $U(N)$. In this case the Higgs branch SCFT is, by the ADHM construction, the same as a sigma model on the moduli space of N instantons in $SU(N_f)$ (on $\mathbb{R}^4$). Note that in this case the problem with the moduli space of the V matrices described in the previous paragraph does not arise, since the positive contribution of the adjoint hypermultiplet to the moduli space metric cancels the negative contribution of the vector multiplet. The action for the adjoint hypermultiplet, which we will denote by X, is of the same form as (2.2):

$$\begin{aligned}\mathcal{L}_H^{(adj)} = \int d^2x \, \mathrm{tr}\Big\{ & (\mathcal{D}_\mu X)^2 + ([X, V])^2 + \bar{\psi}_X \gamma^\mu \mathcal{D}_\mu \psi_X + \\ & + \bar{\psi}_X [V, \psi_X] + \bar{\psi}_V [X, \psi_X] + D[X, X] \Big\}, \end{aligned} \tag{3.11}$$

and the full action of the Higgs branch SCFT will be the sum of (2.2) and (3.11). Now, we expect to get an effective "throat" theory involving both the vector multiplet and the adjoint hypermultiplet fields, since the adjoint hypermultiplet does not become massive for large V.

To derive the effective potential for X in the "throat" region we note that the vector multiplet kinetic term which we obtain by integrating out the hypermultiplets includes (using supersymmetry) a term of the form $N_f \, \mathrm{tr}(D^2)/2V_0^2$. Now, integrating

[7] The moduli space approximation would include only the diagonal elements of the vector multiplet matrices, integrating out the off-diagonal elements.

[8] This is not to be confused with the situation relevant for DLCQ, which we will discuss momentarily. We have so far been discussing the system with only fundamental hypermultiplets. In the DLCQ application there is an additional adjoint hypermultiplet.

out D leads (using (3.11)) to a potential of the form

$$\frac{2}{N_f}\,\mathrm{tr}([X,X]^2)V_0^2\,. \tag{3.12}$$

Combining our results, we find that the bosonic potential of the adjoint fields in configurations close to $V = V_0 I$ is of the form

$$\frac{2}{N_f}V_0^2\left[\mathrm{tr}([\delta V,\delta V]^2)+\mathrm{tr}([\delta V,X]^2)+\mathrm{tr}([X,X]^2)\right]. \tag{3.13}$$

The IR limit of the effective action In the case of the ADHM sigma model, we can proceed further and discuss the low-energy limit of the effective action for large N. In this case, if we discuss the theory on a cylinder, the lowest energy configurations are given by "long string" configurations, which can carry energies proportional to $1/N$ in the large N limit. Since our "throat" theory is a $\mathrm{U}(N)$ gauge theory, with 8 scalar adjoint matrices (4 in V and 4 in X), we can construct such "long string" configurations [33, 34, 35] just like in the DLCQ of type IIA string theory [36, 37, 38, 39], by looking at configurations which change by a non-trivial $\mathrm{U}(N)$ gauge transformation when going around the circle. This relies on the potential term (3.13), which forces the low-energy configurations to sit on the moduli space where all 8 of the matrices commute and can be simultaneously diagonalized (when we fix V_0 and look at configurations with energies of order $1/N$).

We expect that the low-energy effective theory on these "long strings" will be exactly the theory we found in the Higgs branch "throat" for $N = 1$. Since the "long strings" effectively live on a circle which is larger by a factor of N, the mass gap for such configurations is lower by a factor of N than what we found in the $N = 1$ case.[9] We can also compute the leading correction to this effective action, which corresponds to the interaction by which "long strings" split and join. The analysis is a slight variant of the same analysis in the context of the DLCQ of the IIA string [33, 34, 35], and the main idea is still that this interaction is given by a twist operator [35] whose coefficient is determined by dimensional analysis.

In the case of the DLCQ of type IIA string theory, the UV is given by the $1+1$ dimensional $\mathcal{N} = (8,8)$ $\mathrm{U}(N)$ gauge theory. The theory in that case includes interactions of the form $g_{YM}^2\,\mathrm{tr}([X,X]^2)$, where the 8 X matrices have canonical kinetic terms. The low-energy theory is an $(\mathbb{R}^8)^N/S_N$ orbifold, in which the string coupling may be described as a twist operator. The coefficient of this operator can be determined by dimensional analysis; since the operator has dimension 3, its coefficient must scale as $1/g_{YM}$.

In our case, we claim that the effective theory for "long strings" propagating far along the "throat" region is similar, but with $2V_0^2/N_f$ replacing g_{YM}^2. This follows from the effective potential (3.13) for configurations where V is close to $V_0 I$. We conclude that again the correction to the low-energy theory, expanded around the

[9] Recall that we measure energy in units of the inverse size of the circle.

configuration $V \simeq V_0 I$ (which breaks conformal invariance), will be governed by a twist operator in the low-energy orbifold theory, but that this time its coefficient will be proportional to $\sqrt{N_f/2}/V_0$. Note that for small δV we can ignore the fact that the V's really live on $\mathbb{R} \times S^3$ and not on $\mathbb{R}^4$.

3.4 Blowing up the singularities

In the previous subsections we saw how a careful analysis of the behavior of the Higgs branch SCFT near the singularities of the moduli space leads to a simple description involving a continuous spectrum localized near the singularities. Another way to analyze the theory near the singularities, which was used for the $0+1$ dimensional theory in [40], is to blow up the singularities of the Higgs branch by adding Fayet-Iliopoulos (FI) terms of the form $\int d^2x \zeta \, \mathrm{tr}(D)$ to the lagrangean (of course, this is only possible for $\mathrm{U}(N)$ gauge theories, while we expect most of our previous discussion to be more general). The FI term ζ, like D, is a triplet of $\mathrm{SU}(2)_R$, and it is an exactly marginal deformation of the Higgs branch SCFT. Adding such a term lifts the origin of the Higgs branch ($H = 0$), and schematically the minimal allowed value for H now becomes $H \sim \sqrt{\zeta}$. The Coulomb branch is also lifted, so we expect the "throat" in the Higgs branch, which originally connected the two branches, to no longer be infinite. For large N this effect was discussed in [10], here we will see how it works for any value of N.

The effect of ζ on the effective "throat" theory is as follows. As described above, when we integrate out the hypermultiplets we get a one-loop term proportional to $\mathrm{tr}(D^2)/V_0^2$. Now, if we integrate out D, we find a potential of the form $\zeta^2 V_0^2$. This potential prevents states from going to large values of V, as expected. In fact, the leading potential which is generated is precisely the potential appearing in the $\mathcal{N} = 4$ super-Liouville theory [32], as found also in [10]. This potential, which preserves $\mathcal{N} = 4$ superconformal symmetry, can be written using the "throat" variables (for $N_f \geq 3$) in the form $\zeta^2 \int d^2x \, d\theta \, d\overline{\theta} \, \mathcal{O}_{1/2}$, where $\mathcal{O}_j = e^{j\phi\sqrt{2/N_f}} V_{j,j}$ and $V_{j,j}$ are the primaries of the bosonic $\mathrm{SU}(2)$ level $(N_f - 2)$ WZW model. It is easy to check that this deformation is marginal, and it includes in particular a term which is exponential in ϕ and serves as a "Liouville wall".

A similar behavior is expected when we turn on a theta term of the form $\int d^2x \theta \, \mathrm{tr}(F_{01})$, which is related by supersymmetry to the FI terms, but we will not analyze this in detail here. For $\mathrm{U}(1)$ with $N_f = 2$ and $\theta = \pi$ we expect to reproduce the free $\mathbb{R}^4/\mathbb{Z}_2$ orbifold theory [41], but we will not attempt to show this here.

4. The DLCQ of $\mathcal{N} = (2,0)$ "little string theories"

The rest of this paper will involve applying our results concerning the Higgs and Coulomb branch SCFTs to the DLCQ of "little string theories" (LSTs) [11, 12, 13]. The original definition of LSTs with 16 supercharges, as $g_s \to 0$ limits of

NS5-branes or of ALE singularities, is not very useful for making computations in these theories. However, we now have two more explicit descriptions which allow computations, one which is a DLCQ description at finite momentum on a compact light-like circle [2, 3, 18, 19], and the other which is a holographic dual of the LSTs [15] (see also [42, 43], and see [44, 45, 46, 47] for similar holographic duals to LSTs with less supersymmetry). The latter description makes several predictions about LSTs, and we will attempt here to verify one of these predictions in the DLCQ description of the LSTs, using what we learned above about the behavior of the corresponding $1+1$ dimensional field theories. In this section we will analyze the DLCQ of $\mathcal{N}=(2,0)$ supersymmetric LSTs, and in the next section that of $\mathcal{N}=(1,1)$ LSTs.

4.1 Derivation of the DLCQ of $(2,0)$ LSTs

There are two simple derivations of the DLCQ of the A_{k-1} $(2,0)$ LST [2, 3] (we will not discuss the D or E cases here, since they seem to present no new issues). The first uses the definition of the LST as the $g_s \to 0$ limit of the theory on k NS5-branes in type IIA string theory. The DLCQ of type IIA string theory (with momentum $P_- = N/R$) is given by a $1+1$ dimensional $\mathcal{N}=(8,8)$ U(N) gauge theory on a circle. The radius of the circle and gauge coupling are given by (up to numerical constants)

$$\Sigma = \frac{1}{RM_s^2}\,, \qquad g_{YM} = \frac{RM_s^2}{g_s}\,. \tag{4.1}$$

Longitudinal NS5-branes are described by adding hypermultiplets in the fundamental representation [48], and the limit $g_s \to 0$ corresponds to the limit $g_{YM} \to \infty$, or the IR limit of the gauge theory. In this limit, the $\mathcal{N}=(4,4)$ U(N) gauge theory with an adjoint hypermultiplet and k fundamental hypermultiplets flows to two different SCFTs, one describing the Coulomb branch and the other describing the Higgs branch; it is natural to identify the SCFT of the Higgs branch with the DLCQ of the decoupled theory on the NS5-branes.

We can derive the same result also directly in the LST, using Seiberg's description of the DLCQ as equivalent to a compactification on a space-like circle of radius $R_s \to 0$ with appropriate boosts to keep the energy finite [49]. In this description we are led to discuss the A_{k-1} $\mathcal{N}=(2,0)$ "little string theory" on a small space-like circle with N units of momentum on the circle; by T-duality this is equivalent to the A_{k-1} $\mathcal{N}=(1,1)$ "little string theory" on a large space-like circle with N strings wrapped on the circle. In the DLCQ limit we are interested in very low energies in this configuration; then, we can identify the strings as instantons in the low-energy U(k) gauge theory, whose low-energy modes are described by motion on the instanton moduli space, and so we find that the DLCQ is the $1+1$ dimensional sigma model on the moduli space of N U(k) instantons. Using the ADHM construction, this is equivalent to the previous result.

As in the $0+1$ dimensional case [40], we can identify in the DLCQ description all the space-time symmetries which commute with $P_- = N/R$. We will not discuss this in detail here since most identifications are similar to those described in [40]. Let us mention just how the SU(2) symmetries match between the two theories. The $\mathrm{SU}(2)_r \times \mathrm{SU}(2)_l$ symmetry, which is the R-symmetry of the $\mathcal{N}=(4,4)$ superconformal algebra, is identified with the $\mathrm{SO}(4)_R$ global symmetry of the six dimensional LST (which originates from rotations transverse to the branes). The $\mathrm{SU}(2)_R$ symmetry and an additional $\mathrm{SU}(2)_X$ symmetry which is the flavor symmetry of the adjoint hypermultiplet are identified with the SO(4) symmetry rotating the four spatial dimensions transverse to the compact light-like direction. In the $1+1$ dimensional case there is a much larger algebra than that which is required by the DLCQ, including full super-Virasoro and $\mathrm{SU}(2)_r \times \mathrm{SU}(2)_l$ super-Kac-Moody algebras. The space-time role of the additional generators is unclear.

4.2 The "throat" states of the $(2,0)$ LSTs

The "throat" states in the holographic description The holographic description of the A_{k-1} $(2,0)$ LSTs [15], which is given by the near-horizon limit of k type IIA NS5-branes, includes a "throat" region given by a linear dilaton background of type IIA string theory [50]. The full background interpolates between an $AdS_7 \times S^4$ background of M theory and the "throat" region. The string metric in the "throat" region of the holographic description of the A_{k-1} LST is

$$ds^2 = dx^2_{\mathbb{R}^6} + d\phi^2 + \frac{k}{2} d\Omega_3^2 \,. \tag{4.2}$$

There is also a 3-form field $H = -kd\Omega$, and the string coupling behaves as $g_s \propto e^{-\phi\sqrt{2/k}}$, so that the string world-sheet theory in this region is the sum of a free scalar with background charge $\tilde{Q} = -\sqrt{2/k}$, four free fermions and a bosonic level $(k-2)$ SU(2) WZW model (and also six more free scalars and fermions corresponding to the space-time coordinates of the LST). The "throat" contains a continuum (from the six dimensional point of view) of states corresponding to particles (supergravity particles or more general string states) with some momentum (incoming, outgoing or some combination) in the "throat" (ϕ) direction. These states are delta-function normalizable in the "throat", and may presumably be extended to the strong coupling region in such a way that they are still delta-function normalizable in the full theory. Thus, we can take linear combinations of these states that are normalizable. Since these states can be given any longitudinal momentum we want (subject to the mass-shell constraint) we should be able to find these states also in the DLCQ description.

The spectrum of chiral operators in the "throat" region includes one chiral multiplet for every primary field of the corresponding bosonic SU(2) WZW theory.[10]

[10]For the G LST, where G is a group of ADE type, we have the corresponding SU(2) modular invariant.

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The lowest multiplet is the supergraviton multiplet. For this multiplet the on-shell condition is of the form

$$E^2 - p^2 - \tilde{q}(\tilde{q} + i\tilde{Q}) = 0\,, \tag{4.3}$$

where E is the energy, p (a 5-vector) is the spatial momentum and $\tilde{q}$ is the momentum in the ϕ direction (i.e. the vertex operator is $e^{i\tilde{q}\phi}$), all dimensionless and measured in string units. The S-matrix for these states was discussed in [51].

In the DLCQ of this theory, we should find such states with longitudinal momentum $p_- = N/R$ and with any $\tilde{q}$ and transverse momentum $\vec{p}$. The DLCQ hamiltonian p_+ for these states should be of the form

$$p_+ = \frac{(\vec{p}^2 + \tilde{q}(\tilde{q} + i\tilde{Q}))}{p_-} = \frac{R(\vec{p}^2 + \tilde{q}(\tilde{q} + i\tilde{Q}))}{N}\,. \tag{4.4}$$

The minimal value of this hamiltonian, which occurs for $\vec{p} = 0$ and $\tilde{q} = -i\tilde{Q}/2$, is given by $R\tilde{Q}^2/4N = R/2Nk$. Similar equations (with a larger mass gap) arise for the other chiral multiplets.

In addition to the spectrum, we also know how the string interactions are supposed to behave like in the "throat". The linear dilaton causes the string coupling to behave as $g_s \propto e^{\tilde{Q}\phi}$, and we should be able to reproduce this behavior in the DLCQ.

The "throat" states in the DLCQ description Matching the spectrum and string interactions in the "throat" region between the DLCQ and the holographic description is straightforward in view of the field theoretic analysis in sections 2 and 3. As an initial step it is interesting to compare the Higgs branch scaling of V with the near-horizon scaling of the coordinates [4]. In DLCQ the lagrangean for the coordinates r^i transverse to the brane is naturally of the form

$$\mathcal{L}_{DLCQ} \sim \frac{1}{R\Sigma}\int d^2x(\partial_\mu r^i)^2\,, \tag{4.5}$$

corresponding to a low-energy quantum-mechanical hamiltonian proportional to R. The relation to the SYM lagrangean (2.1) variables is thus given by

$$V^i = \frac{g_{YM} r^i}{\sqrt{R\Sigma}} = R\left(\frac{M_s^3}{g_s} r^i\right). \tag{4.6}$$

The quantity in parenthesis is precisely what we keep fixed in the near-horizon limit [42, 15] of NS 5-branes.[11] Hence, keeping the dimension one V fixed is equivalent to taking the near-horizon limit.

We begin our discussion of the "throat" states in the DLCQ description with the simplest case of the A_{k-1} LST with $N = 1$. The DLCQ is then simply the Higgs branch SCFT of the U(1) gauge theory with $N_f = k$ hypermultiplets, discussed

[11] It is the tension of the strings that originate from stretched D2-branes.

earlier. In addition there is also an adjoint hypermultiplet, which in the U(1) case is just a free hypermultiplet, giving rise to states of momentum $\vec{p}$ with energy $\vec{p}^2$. We found above that this theory has a region in moduli space which looks like a "throat", involving a supersymmetric level k SU(2) WZW model and a scalar field with background charge $Q = (k-1)\sqrt{2/k}$. We also found a zero-point energy in these variables, so that the hamiltonian (3.6) is of the form $H_{DLCQ} = (1/\Sigma)(\vec{p}^2 + q(q+iQ) + 1 - k/2)$, where we reintroduced the correct units for the hamiltonian by using the radius of the circle in the SCFT, which is $\Sigma = 1/RM_s^2$. As described at the end of section 3.1, we can define $\tilde{q} = q + i(Q - \tilde{Q})/2$ (where $\tilde{Q} = -\sqrt{2/k}$ as above), and rewrite this hamiltonian as $H_{DLCQ} = RM_s^2(\vec{p}^2 + \tilde{q}(\tilde{q} + i\tilde{Q}))$. This is exactly the same equation we found above for $p_+ = H_{DLCQ}$ (for $N = 1$). Thus, we find the same continuum of "supergraviton" states in both descriptions of the LSTs. In section 3 we described the change of variables from q to $\tilde{q}$ in the context of relating the Higgs branch and Coulomb branch "throats" of the same theory; here we see that the same change of variables relates the DLCQ Higgs branch variables and the space-time variables, even though there is no direct relation (in the DLCQ context) between the space-time "throat" and the Coulomb branch after we take the decoupling limit. We can easily generalize this analysis to the other chiral states of the LSTs. In both descriptions they are in a one-to-one correspondence with the primaries of the (same) bosonic SU(2) WZW model, and it is easy to check that we find exactly the same states in both cases.

For higher N the situation is similar, for states localized near some position in the "throat" which are the configurations we discussed in section 3.2. As in the DLCQ of critical string theories, we need to look at "long string" configurations that can carry energies proportional to $1/N$ in the large N limit. We have discussed in section 3 how to go to the "long strings" for large N and what is the coefficient of the twist operator in the effective theory. Translating the results there to the language of the current section leads to exactly the correct hamiltonian for the "throat" states, and to a string coupling in space-time of the form

$$g_s \propto \frac{1}{V_0} \simeq e^{-\phi\sqrt{2/k}}. \tag{4.7}$$

This is exactly the same string coupling found in the holographic description of the "little string theories" [15] (note the factor of $\sqrt{2}$ difference in the normalization of ϕ between our paper and [15]). Thus, we see that both the low-energy states (in the "throat") and their interactions are correctly reproduced in the DLCQ.

4.3 Blowing up the singularities in the DLCQ

In section 3.3 we discussed blowing up the singularities in the Higgs branch by turning on FI terms (or theta angles) in the gauge theories, and we showed that this turns on a "Liouville wall" in the "throat" which prevents states from propagating deep into the "throat" (this description is useful for small FI terms, when the "throat" approximation is still valid near the "wall"). In the DLCQ context we can give this blow-up a space-time interpretation. Following the derivation of the DLCQ, using the definition of the LST as the $g_s \to 0$ limit of NS5-branes, we can show that the FI terms correspond in this definition to turning on 3-form RR fields parallel to the NS5-branes (which are non-trivial, despite having zero field strength, due to their interaction with the 3-form fields on the NS5-branes). This is a generalization of the analogous deformation of the $(2,0)$ six dimensional SCFT discussed in [40, 52] (and more precisely in [53]), and of the deformations used for constructing non-commutative Yang-Mills theories.

Presumably, the decoupling limit of the NS5-branes in the presence of this RR field leads to some analog of a non-commutative "little string theory". Since we identified the "throat" of the Higgs branch with the linear dilaton region in the holographic dual of the LSTs, we see that turning on the deformation turns on a wall that prevents states from going into the weak coupling region in space-time; thus, we expect this deformation in the holographic description to leave the $AdS_7 \times S^4$ region and the strong coupling part of the linear dilaton region, but to lift the weak coupling region (this is opposite from the standard "Liouville wall"). This should be visible also in the appropriate near-horizon limit of the NS5-brane with RR fields (generalizing the results of [54, 55]). This is opposite to the deformations recently discussed in [46]; they discussed deformations which change the IR behavior of the LSTs, while the deformation discussed here does not change the IR behavior but seems to significantly change the UV behavior. It would be interesting to study these theories further.

5. The DLCQ of $\mathcal{N} = (1,1)$ "little string theories"

5.1 Derivation of the DLCQ of $(1,1)$ LSTs

To derive the DLCQ of the A_{k-1} $(1,1)$ LSTs, it is simplest to start from their definition as the $g_s \to 0$ limit of the theory on an A_{k-1} singularity in type IIA string theory. In [18, 19] it was shown that the DLCQ description of this is given by the low-energy theory of D-strings near a similar singularity in type IIB string theory.[12] Unfortunately, there is no simple description of this theory. However, if we deform the singularity to the $\mathbb{R}^4/\mathbb{Z}_k$ orbifold, then the theory on the D-strings is [56] the $\mathcal{N} = (4,4)$ $\mathrm{U}(N)^k$ SQCD theory with bi-fundamental hypermultiplets for adjacent group factors (when we arrange the gauge groups on a circle). At the orbifold point the gauge couplings of all $\mathrm{U}(N)$ factors are equal and they are inversely proportional

[12] To derive this most directly, we start with this theory on a small space-like circle with N units of momentum (and with finite g_s); it is natural to T-dualize this to type IIB string theory (with the same singularity) on a large circle with N fundamental strings, but now we get a large coupling for the type IIB string theory so it is natural to S-dualize, and we end up with N D-strings near an A_{k-1} singularity at weak coupling. The $g_s \to 0$ limit is again the low-energy ($g_{YM} \to \infty$) limit.

to the original type IIA string coupling. Thus, the $g_s \to 0$ limit again corresponds to taking the IR limit of this theory, which flows to two decoupled SCFTs. In this case it is the Coulomb branch theory which we identify with the DLCQ of the LST, since the Higgs branch describes the motion of the strings away from the singularity.

One still needs to discuss the effects of the deformation from the A_{k-1} singularity to the non-singular $\mathbb{R}^4/\mathbb{Z}_k$ orbifold. This deformation corresponds to turning on specific non-zero B fields on the various vanishing 2-cycles of the singularity. If we follow the chain of dualities, we find that in the original description turning on these B fields corresponds to turning on a longitudinal Wilson line in the low-energy $\mathrm{U}(k)$ gauge theory.[13] More precisely, the orbifold point corresponds to a Wilson line of the form

$$RA_- = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{k} & 0 & \cdots & 0 \\ 0 & 0 & \frac{2}{k} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{k-1}{k} \end{pmatrix}. \tag{5.1}$$

This Wilson line has the effect of shifting the longitudinal momentum of charged particles. The fields in the Cartan subalgebra (corresponding to diagonal matrix elements) still have integer longitudinal momentum, but the other fields now have a fractional momentum; the space-time fields (e.g. W bosons) coming from the (i,j) element of a $\mathrm{U}(k)$ matrix have $RP_- = (i-j)/k \pmod 1$.

Changing the value of this Wilson line corresponds to changing the B fields in the A_{k-1} singularity used to define the DLCQ theory. In the field theory of the D-strings, this corresponds to changing the ratios between the Yang-Mills couplings of the k $\mathrm{U}(N)$ gauge groups. In particular, turning off the Wilson line (or some components of it, leading to an unbroken low-energy non-abelian gauge group in space-time) corresponds to making ratios of gauge couplings infinite. At first sight, since to describe the LST we are taking the gauge couplings to infinity, these ratios of gauge couplings seem to be unimportant; however, we will see below that they show up in the moduli space metric.

The identification of the SU(2) symmetries between the DLCQ field theory and the space-time LST is the following. The $\mathrm{SU}(2)_r \times \mathrm{SU}(2)_l$ group in the two dimensional gauge theory is now identified with the longitudinal SO(4) of the six dimensional theory in the light-cone frame, while $\mathrm{SU}(2)_R$ is identified with a diagonal subgroup of the global $\mathrm{SO}(4)_R$ symmetry group of the LST. As discussed above, we expect that in the Coulomb branch SCFT the $\mathrm{SU}(2)_R$ symmetry will be enhanced to an $\mathrm{SU}(2) \times \mathrm{SU}(2) \simeq \mathrm{SO}(4)_R$ Kac-Moody algebra, although we cannot exhibit this symmetry directly in the gauge theory.

[13]The B fields correspond to the longitudinal components of the RR vector fields in the twisted sector of the $\mathbb{R}^4/\mathbb{Z}_k$ orbifold, which are the Cartan subalgebra of the low-energy $\mathrm{U}(k)$ theory.

5.2 The low energy theory

The metric on the moduli space The moduli space of the Coulomb branch is given by the scalars V in the $\mathrm{U}(N)^k$ vector multiplets; the four scalars of each $\mathrm{U}(N)$ factor are commuting on the moduli space, so they can be simultaneously diagonalized. We can thus parametrize the moduli space by the vectors $\vec{V}_i^{(j)}$, where V is a 4-vector, $i = 1, \ldots, k$ labels the different $\mathrm{U}(N)$ groups, and $j = 1, \ldots, N$ labels the eigenvalues of the corresponding matrices (in some order). The moduli space metric gets contributions only from tree level and 1-loop. Thus, following [9], it is easy to compute the exact metric, which is

$$ds^2 = \frac{1}{g_{YM}^2} \sum_{i=1}^{k} \sum_{j=1}^{N} (d\vec{V}_i^{(j)})^2 + \sum_{i=1}^{k} \sum_{j,l=1}^{N} \frac{d(\vec{V}_i^{(j)} - \vec{V}_{i+1}^{(l)})^2}{|\vec{V}_i^{(j)} - \vec{V}_{i+1}^{(l)}|^2} - \sum_{i=1}^{k} \sum_{j=1}^{N-1} \sum_{l=j+1}^{N} \frac{2d(\vec{V}_i^{(j)} - \vec{V}_i^{(l)})^2}{|\vec{V}_i^{(j)} - \vec{V}_i^{(l)}|^2}, \tag{5.2}$$

where $(i+1)$ is taken modulo k.

In the DLCQ of the LST we are interested in the limit $g_{YM} \to \infty$, so naively the tree level term drops out; however, as discussed in section 2, we should also take the V's to infinity such that $\Phi \sim V/g_{YM}$ remains constant in the IR limit. This can be seen in various ways. We justified this in section 2 by requiring zero dimension for the moduli space variables, but in the DLCQ context we can also see this by noting that the space-time distance between states is proportional to Φ and not to V. It will be convenient to normalize $\Phi \equiv \sqrt{k}V/g_{YM}$ (because g_{YM} in this theory is actually $\sqrt{k}$ times what it was for the D-strings without the orbifold), and then we find that the actual metric which is relevant for the DLCQ of the LST is of the form

$$ds^2 = \frac{1}{k} \sum_{i=1}^{k} \sum_{j=1}^{N} (d\vec{\Phi}_i^{(j)})^2 + \sum_{i=1}^{k} \sum_{j,l=1}^{N} \frac{d(\vec{\Phi}_i^{(j)} - \vec{\Phi}_{i+1}^{(l)})^2}{|\vec{\Phi}_i^{(j)} - \vec{\Phi}_{i+1}^{(l)}|^2} - \sum_{i=1}^{k} \sum_{j=1}^{N-1} \sum_{l=j+1}^{N} \frac{2d(\vec{\Phi}_i^{(j)} - \vec{\Phi}_i^{(l)})^2}{|\vec{\Phi}_i^{(j)} - \vec{\Phi}_i^{(l)}|^2}. \tag{5.3}$$

As mentioned above, if we change the longitudinal Wilson line in space-time, we need to change the ratios of Yang-Mills couplings in the DLCQ description; we see that in the moduli space metric this corresponds to having different coefficients for the $d\vec{\Phi}^2$ terms corresponding to different gauge groups (the coefficients in the normalization we chose are exactly the values of the B-fields in a particular basis for the 2-cycles). In particular, if we try to turn off some of the Wilson line components, some of the $d\vec{\Phi}^2$ terms vanish, leaving only the 1-loop contributions (and changing the asymptotic form of the moduli space).

The singularities In the presence of the Wilson line, the metric is flat when all the eigenvalues are far from each other, but develops various singularities when some of the eigenvalues come close together. When a vector $\vec{\Phi}_i^{(j)}$ approaches a vector $\vec{\Phi}_{i+1}^{(l)}$ the theory looks like a $\mathrm{U}(1)$ theory with one charged hypermultiplet. Naively this theory has a "throat"-like singularity, but in fact the moduli space description in the "throat" breaks down [9], and there is no good description of this singularity. We do not expect to have a "throat" emerging from this sort of singularity; at least, there is no other branch emanating from such a point (before taking the IR limit) which would indicate such a "throat". It is believed that this singularity is smoothed out [9, 57].

A hint to the way by which this singularity is smoothed out is given by the fact that at low energies (i.e restricting to the quantum mechanics of the zero-modes[14]) it is known that there is a bound state living at this singularity (the D0-D4 bound state). This suggests that there is also a ground state of the $1+1$ dimensional theory which is localized near the singularity (this was referred to as a "quantum Higgs branch" in [3]). We will later motivate this expectation using DLCQ considerations.

Additional singularities arise when several vectors $\vec{\Phi}^{(l_n)}_{i+n}, n = 0, \ldots, j$, $(j < k-1)$ approach each other. There is no good description of these singularities, but by analogy with the previous case we would expect the moduli space description to break down, and the existence of a bound state at low energies (again, this expectation will be supported by the DLCQ analysis below).

Another type of singularity occurs when a vector $\vec{\Phi}^{(j)}_i$ approaches a different vector $\vec{\Phi}^{(l)}_i$. In this case the effective theory is like the pure SU(2) theory, for which again the moduli space description breaks down since the metric becomes negative [9]. Also in this case it is believed that there is no "throat" emanating from such a singularity, since there is no other branch coming out of it prior to taking the low-energy limit. Presumably, this singularity is also smoothed out in appropriate variables.

A more serious singularity arises when k vectors coming from different gauge groups approach each other; in this case we expect a real "throat" to develop in the moduli space (as is evident in the metric), since such a singularity connects (before taking the IR limit) to the Higgs branch of the theory, in which the hypermultiplets acquire VEVs. We will discuss this "throat" in section 5.3.

DLCQ applications For now, let us return to the question of the low-energy states. We will analyze them in detail for $N = 1$; the analysis of other cases is similar for configurations where the different eigenvalues of each matrix are far from each other (giving N copies of the $N = 1$ case), and is not clear for other configurations.

The simplest configuration involves slow motion on the asymptotic region of the moduli space, where all k of the $\vec{\Phi}$'s are far from each other and the metric is flat. Restricting to the zero modes on the circle, we identify this configuration with a space-time configuration involving k W-bosons,[15] coming from the $(1,2), (2,3), \ldots, (k-1, k)$ and $(k, 1)$ elements of the low-energy U(k) matrices. Each vector $\vec{\Phi}_i$ may be iden-

[14]We assume that supersymmetry protects the zero-mode dynamics from being renormalized by integrating out the non-zero momentum modes.

[15]We use the term W-boson to denote any element of the space-time vector multiplet. The precise element in space-time will be determined by the fermion zero modes.

tified with the transverse position of one of these particles, each of which carries $1/k$ units of longitudinal momentum (such that the total longitudinal momentum is $P_- = 1/R$), and the hamiltonian is consistent with this interpretation.[16] If we change the longitudinal Wilson line, the description remains the same; the coefficients of the $d\vec{\Phi}^2$ terms in the moduli space metric now change in the asymptotic region, corresponding to the different longitudinal momenta now carried by the k particles, and we still get an exact agreement between the space-time and DLCQ theories. If we try to turn off a component of the Wilson line we see that this flat region of the metric goes off to infinity, as we expect since then one of the particles we describe has $P_- \to 0$ and we would not expect to describe it as a simple particle in the DLCQ. From here on we will discuss only the "maximal" Wilson line (5.1).

We still need to account for the rest of the W-bosons in space-time. Our proposal is that these are described by bound states which are the same bound states discussed above in the context of resolving the singularities in the moduli space. For example, the W-boson corresponding to $(1,3)$ elements of the matrices may be viewed as a bound state at threshold of the $(1,2)$ and $(2,3)$ W-bosons, so we identify it with a configuration in which two adjacent vectors, say $\vec{\Phi}_1$ and $\vec{\Phi}_2$, come together and form a bound state in the region where naively there is a "throat". From the space-time description we see that the effective dynamics of this bound state should be (when it is far from the other states) just a sigma model on $\mathbb{R}^4$, with a metric whose coefficient is twice that of the original two vectors. This would reproduce the space-time dynamics of a particle with $RP_- = 2/k$, as desired. Similarly, by taking various combinations of such bound states, we can describe any other configuration of W-bosons whose total momentum is $RP_- = 1$.

From this description it is clear that states corresponding to photons (vectors in the Cartan subalgebra), which have $RP_- = 1$, live in the "throat" region, where all the eigenvalues come together. We will discuss this region in the next subsection.

5.3 The "throat" states of the $\mathcal{N} = (1,1)$ LST

In the $\mathcal{N} = (1,1)$ case the Coulomb branch metric receives one-loop corrections that cause it to include semi-infinite "throats", so it seems that it may be possible to identify the "throat states" more directly than in the $\mathcal{N} = (2,0)$ case discussed above. However, surprisingly, it actually seems more complicated to obtain a precise understanding of the "throat states" in this case.

There is one case in which we can precisely identify the "throat states", which is the A_1 $\mathcal{N} = (1,1)$ LST with one unit of longitudinal momentum. From the holographic description [15] we learn that in this case there is (since the bosonic

[16] A related discussion of W-boson states appears in [19].

WZW model in the space-time string theory has level $k_{\mathrm{SU}(2)} = 2 - 2 = 0$) just one chiral multiplet of states propagating in the "throat", which is the graviton multiplet. The scalar ϕ parametrizing the "throat" coordinate has a background charge of $\tilde{Q} = -\sqrt{2/k} = -\sqrt{2/2} = -1$. The on-shell condition (or the condition for the corresponding vertex operator to be of dimension $(1,1)$) for these states is $E^2 - p^2 - \tilde{q}(\tilde{q} + i\tilde{Q}) = 0$, where p (a 5-vector) is the spatial momentum and $\tilde{q}$ is the momentum in the ϕ direction (i.e. the vertex operator is $e^{i\tilde{q}\phi}$), all measured in string units.

For $N = 1$ we can find exactly the same states in the DLCQ. The DLCQ description of this theory is the IR limit of the Coulomb branch of the $\mathrm{U}(1)\times\mathrm{U}(1)$ $\mathcal{N} = (4,4)$ gauge theory with 2 bifundamental hypermultiplets, which is equivalent to a free $\mathrm{U}(1)$ gauge theory (which is just a sigma model on $\mathbb{R}^4$) and another $\mathrm{U}(1)$ theory with two charged hypermultiplets. As described in [9], when we integrate out these hypermultiplets the metric develops a "throat". In the "throat", the theory of $\mathrm{U}(1)$ with $N_f = 2$ is a single boson ϕ with background charge $\tilde{Q} = -\sqrt{2/N_f} = -\sqrt{2/2} = -1$, and (after the chiral twist of the fermions) a bosonic WZW model at level $k_{\mathrm{SU}(2)} = 2-2 = 0$ (there are also some free fermions). Thus, in the DLCQ we can construct states (localized in the "throat" region) with an arbitrary transverse 4-momentum $\vec{p}$ (from the free $\mathrm{U}(1)$ part) and an arbitrary ϕ momentum $\tilde{q}$. Such a state will have a DLCQ hamiltonian of the form

$$H_{DLCQ} = R[\vec{p}^2 + \tilde{q}(\tilde{q} + i\tilde{Q})]\,, \tag{5.4}$$

so that

$$p_+p_- - \vec{p}^2 = H_{DLCQ} * \frac{1}{R} - \vec{p}^2 = \tilde{q}(\tilde{q} + i\tilde{Q}), \tag{5.5}$$

which is exactly the same answer we found above from the holographic description. Thus, in this case we have exactly the right "throat states" (the fermion zero modes give these states the correct multiplicities in space-time). Note that in space-time the dilaton becomes smaller as we go out in the "throat" (away from the 5-branes), while in the DLCQ description it becomes larger as we go into the "throat" (ϕ in the DLCQ is $(-\phi)$ in space-time), but there is no obvious relation between the two dilatons (as discussed above, the string interactions in space-time are related to a different world-sheet operator).

For higher values of N and/or k things become much more complicated. Let us start by discussing higher k. In this case, ignoring the free $\mathrm{U}(1)$ multiplet, the $4k-4$ dimensional moduli space has a co-dimension $4k-4$ singularity at which, as described above, the metric seems to develop a semi-infinite "throat". However, for $k > 2$ it is not clear how to go to a simpler description of the "throat" theory as we did for $k = 2$. We expect to still have one coordinate labeling the distance "down the throat", but it is not clear what would be a good description of the $4k - 5$ angular variables in this "throat" (or whether some of them decouple). One expects these variables to be equivalent to a level $(k-2)$ bosonic SU(2) WZW model, corresponding to the "throat" theory we have in space-time (similar to what we found for the $(2,0)$ LST). However, except for "answer analysis" and the fact that this description has the correct $\mathrm{SU}(2)\times\mathrm{SU}(2)$ symmetries (which we have in space-time, as discussed in the next paragraph), there is no good argument for it. Encouraged by the clean picture that arises in the $(2,0)$ case, we will view the "throat" states of the holographic description as a prediction about the behavior of "throat" states in the Coulomb branch of these gauge theories.

The SU(2) that is part of the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ symmetry of the putative SU(2) WZW model in the effective theory of the "throat" is what we called before $\mathrm{SU}(2)_R$. Since this group acts only on fermions in the Coulomb branch, the WZW model cannot come directly from the bosonic moduli space coordinates, but must involve the fermionic variables. Presumably, again the appropriate "throat" variables are some combinations of the fermions. By now this is not surprising, since this is how we got the "throat" WZW model in the Higgs branch, and similar "bifermionic coordinates" were found in the DLCQ of $\mathcal{N}=(2,0)$ SCFTs [4] and in instanton computations on $AdS_5\times S^5$ (see [5] and references therein).

For higher values of N we again expect to find "long string" configurations which can be identified with the strings propagating in the "throat" of the holographic description. Since on the Coulomb branch we have a $\mathrm{U}(N)^k$ gauge symmetry it is easy to construct "long string" configurations which vary by a gauge transformation when going around the circle, but we do not know how to show that they are described by the correct theory and that they have the correct interactions (as we showed for the $\mathcal{N}=(2,0)$ case above).

6. Summary and discussion

In this paper we analyzed the superconformal theories arising as the low-energy limit of $\mathcal{N}=(4,4)$ gauge theories. We wrote down an explicit lagrangean for the Higgs branch SCFT, and showed how it leads on one hand to the description of this SCFT as the sigma model on the Higgs branch moduli space, and on the other hand to "throat" states localized near the singularities of this space. We analyzed these "throat" states in detail for the cases of U(1) and $\mathrm{U}(N)$ gauge theories, and matched them in the DLCQ context to "throat" states of the $(2,0)$ LSTs. It would be interesting to generalize this analysis to higher order terms and to different gauge groups. Our analysis of the U(1) theory with two hypermultiplets applies to the $\mathbb{R}^4/\mathbb{Z}_2$ sigma model with zero theta angle. A similar analysis of $\mathbb{R}^4/\mathbb{Z}_k$ singularities requires studying $\mathrm{U}(1)^k$ theories with bifundamental matter, which we leave for further study. We could not write down an explicit lagrangean for the Coulomb branch SCFT, but we wrote down its moduli space metric and used it to analyze some states in the SCFT in section 5.

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The Coulomb and Higgs branch SCFTs of the same gauge theory both contain (when they both exist) isomorphic "throat" regions, though the full theories are quite different and have different asymptotic regions. The "throat" theory has a large $\mathcal{N} = 4$ algebra, and different small $\mathcal{N} = 4$ subalgebras of this are realized as the superconformal symmetries of the Higgs and Coulomb branch theories.

It is important to note that the "throat" states generally account for just a small fraction of the density of states in the Higgs branch SCFT. For example, in the case of U(1) with N_f hypermultiplets, the density of states of the full theory is governed by the central charge $c = 6(N_f - 1)$, while the density of "throat" states is that of a $c = 6$ theory (the "throat" theory in itself does not obey Cardy's formula for the asymptotic density of states). This may be related to the fact that in the "throat" we can only see states that are invariant under the SU(N_f) flavor symmetry of the Higgs branch theory.

In the DLCQ context the density of states in the Higgs branch SCFT (for the case of U(N) gauge group with an adjoint and k fundamental hypermultiplets) is related to the density of states in the A_{k-1} LSTs at high energy densities [58], which in turn is related to the density of states of CGHS black holes [59, 14]. The relevant states in the Higgs branch SCFT are those whose energy scales as $1/N$ in the large N limit. As discussed in [58], in order to reproduce the black hole entropy one needs the density of these states to behave like that of a conformal theory with $c = 6k$. In the sigma model description of the Higgs branch theory it is hard to see states with an energy scaling as $1/N$; however, we saw in section 3 that in the "throat" region we have states whose energy scales like $1/N$, which are the "long string" states. Unfortunately, as discussed above, the density of these states is much smaller (for $k > 2$) than the full density of states which we need to reproduce the black hole entropy. We predict that appropriate states exist in the full Higgs branch SCFT, perhaps corresponding to states with non-trivial flavor SU(k) transformations, but we do not know how to exhibit them directly. It would be interesting to find these states and try to use the DLCQ description to learn about the high-energy behavior of the LSTs.

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